Higgs Mass from g-2 in Anomaly Mediated Supersymmetry Breaking Models

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Abstract

We estimate the upper bound of the Higgs mass and allowed parameter region in the anomaly mediated supersymmetry breaking models. There has been a difficulty that the parameter region cannot be determined precisely in almost SUSY models. However, we succeed to give strict constraints on the parameters from the recent results of the precision measurements of the muon g-2 (anomalous magnetic dipole moment). Especially, the upper bound of the Higgs mass is obtained as a function of only $\tan\beta$, and is estimated less than about 123 (GeV) for all the range of $\tan\beta < 60$.

1 Introduction

The supersymmetry (SUSY) is one of the most attractive extension beyond the standard model (SM). It is important to detect the signals of SUSY not only in the high-energy region but also in low-energy experiments.

In the high-energy physics, finding Higgs bosons is one of the main objectives for both of the SM and SUSY. As a common feature of SUSY models [1, 2], the Higgs boson mass is smaller than one in the SM. Thus, the search for the light neutral Higgs boson is regarded as a decisive test to give an evidence for SUSY that can be performed at present or in the next generation of high-energy colliders. The SM Higgs[1] mass is currently restricted by LEP [3, 4] as follows:

$$m_h^{SM} > 114.4 \text{ (GeV)}. \tag{1}$$

On the other hand, the MSSM lightest neutral Higgs is constrained as [4, 5]:

$$m_h > 90.1 \text{ (GeV)}. \tag{2}$$

m_h has been expected less than about 130 (GeV) in many SUSY models theoretically [1, 2, 6]. However, some of the parameters in such SUSY models have been assumed loosely in almost analyses. In this paper, we show the allowed region of the parameters in more reliable way to be consistent with the low-energy precise physics such as the measurements of the muon anomalous magnetic dipole moment.

The anomalous magnetic dipole moment (g-2) of the muon, defined as $a_\mu = (g_\mu - 2)/2$ for convention, is one of the most precisely measured quantities in the low-energy particle physics. The world sum average, dominated by the recent measurements of the Muon (g-2) Collaboration at Brookhaven National Laboratory [7], reads,

$$a_\mu^{exp} = 11659203 \pm 7 \times 10^{-10} \ . \tag{3}$$

The difference between the experiment and the theoretical prediction in the SM of the muon g-2 is currently [8],

$$a_\mu^{exp} - a_\mu^{SM}(e^+e^-) = (35.5 \pm 11.7) \times 10^{-10} \ , \tag{4}$$

$$a_\mu^{exp} - a_\mu^{SM}(\tau) = (10.3 \pm 10.7) \times 10^{-10} \ , \tag{5}$$

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[1]the SUSY Higgs bosons are described as merely "Higgs" denoted by m_h, in contrast to the Higgs boson in the standard model m_h^{SM} is expressed as "the SM Higgs" throughout this paper.

where, e^+e^- and τ mean the kinds of the hadronic contributions to the muon g-2 based on e^+e^- and τ cross sections, respectively. This inconsistency between e^+e^- and τ-based values stems from their different cross sections [9]. As Eqs.(4) and (5) imply, the muon g-2 in the SM is slightly but significantly lower than the experimental results [8]. Thus, we think of all the 10×10^{-10} scale difference of a_μ between the experimental results of the muon g-2 and its theoretical prediction in the SM,

$$a_\mu^{exp} - a_\mu^{SM} = a_\mu^{SUSY} \, , \tag{6}$$

as the contributions of SUSY effects, in this paper.

As a realistic SUSY model, the anomaly mediated supersymmetry breaking (AMSB) scenario [10] is adopted in this paper. This scenario is a mechanism of SUSY breaking via the super-Weyl anomaly. The AMSB models have four important parameters { $m_{3/2}$, m_0, $\tan\beta$, $\mathrm{sgn}(\mu)$ }, where, $m_{3/2}$ stands for the gravitino mass corresponding to the vacuum expectation value (VEV) of the auxiliary field for breaking SUSY at near the Planck scale. m_0 denotes the bulk mass and introduced to avoid the sleptons being tachyonic. $\tan\beta$ is defined as the ratio between VEVs of the two Higgs doublets as usual. $\mathrm{sgn}(\mu)$ means the sign of the Higgs mixing parameter. For more detail of AMSB models, see [10].

The main purpose of this paper is to estimate the upper bound of the Higgs mass by restricting the allowed parameter region in AMSB models from precisely measured muon g-2.

2 Mass Spectra

In AMSB models, the soft SUSY breaking masses in the diagonal parts of the mass matrices of each scalar particles are given as follows [10, 11, 12] :

$$m_{\tilde{Q}_i}^2 = c_Q m_0^2 + \left(-\frac{11}{50}\alpha_1^2 - \frac{3}{2}\alpha_2^2 + 8\alpha_3^2 + \beta_t\delta_{i,3} + \beta_b\delta_{i,3} \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(L-squarks)} \, , \tag{7}$$

$$m_{\tilde{U}_i}^2 = c_U m_0^2 + \left(-\frac{88}{25}\alpha_1^2 + 8\alpha_3^2 + 2\beta_t\delta_{i,3} \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(u-type R-squarks)} \, , \tag{8}$$

$$m_{\tilde{D}_i}^2 = c_D m_0^2 + \left(-\frac{22}{25}\alpha_1^2 + 8\alpha_3^2 + 2\beta_b\delta_{i,3} \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(d-type R-squarks)} \, , \tag{9}$$

$$m_{\tilde{L}_i}^2 = c_L m_0^2 + \left(-\frac{99}{50}\alpha_1^2 - \frac{3}{2}\alpha_2^2 + \beta_\tau\delta_{i,3} \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(L-sleptons)} \, , \tag{10}$$

$$m_{\tilde{E}_i}^2 = c_E m_0^2 + \left(-\frac{198}{25}\alpha_1^2 + 2\beta_\tau\delta_{i,3} \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(R-sleptons)} \, , \tag{11}$$

$$m_{h_u}^2 = c_{h_u} m_0^2 + \left(-\frac{99}{50}\alpha_1^2 - \frac{3}{2}\alpha_2^2 + 3\beta_t \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(u-type Higgs)} \, , \tag{12}$$

$$m_{h_d}^2 = c_{h_d} m_0^2 + \left(-\frac{99}{50}g_1^4 - \frac{3}{2}g_2^4 + 3\beta_b + \beta_\tau \right) \frac{m_{3/2}^2}{16\pi^2} \qquad \text{(d-type Higgs)} \, , \tag{13}$$

with

$$\beta_t = \alpha_t \left(-\frac{13}{15}\alpha_1 - 3\alpha_2 - \frac{16}{3}\alpha_3 + 6\alpha_t + \alpha_b \right) \, , \tag{14}$$

$$\beta_b = \alpha_b \left(-\frac{7}{15}\alpha_1 - 3\alpha_2 - \frac{16}{3}\alpha_3 + \alpha_t + 6\alpha_b + \alpha_\tau \right) \, , \tag{15}$$

$$\beta_\tau = \alpha_\tau \left(-\frac{9}{5}\alpha_1 - 3\alpha_2 + 3\alpha_b + 4\alpha_\tau \right) \, , \tag{16}$$

where, the index i indicates the generation. $\alpha_n = g_n^2/4\pi$ (n=1, 2, and 3) denotes the gauge coupling constant, and $\alpha_f = y_f^2/4\pi$ (f=t, b, and τ) stands for the Yukawa coupling constant of top, bottom, and tau, respectively. $\delta_{i,j}$ is the Kronecker's delta. m_0 is the bulk mass introduced as the non-anomaly mediated contribution in order to keep the squared slepton masses positive.

In general, the contributions of the bulk mass are not universal among the particles, therefore, the coefficients c_Q, c_U, c_D, c_L, c_E, c_{h_u}, and c_{h_d} can be different from one another. However, in the *minimal* AMSB (mAMSB) model [10], they are universal and normalized as:

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$$c_Q = c_U = c_D = c_L = c_E = c_{h_u} = c_{h_d} = 1 \ . \tag{17}$$

The mass relations, Eq.(7) to (13), are Renormalization Group (RG) invariant except the bulk mass terms proportional to m_0. Since the smaller Yukawa couplings are neglected, the bulk mass terms are running only in the third generation. When $\tan \beta$ is enough small, only the top Yukawa coupling is effective. However, if $\tan \beta$ is large, not only the top but also the bottom Yukawa coupling is influential. Thus, we put both of these two contributions into the RG part of the third generation of the mass relations [12]:

$$\frac{d}{dt} \begin{pmatrix} \delta m_{h_u}^2 \\ \delta m_{U_3}^2 \\ \delta m_{Q_3}^2 \\ \delta m_{D_3}^2 \\ \delta m_{h_d}^2 \end{pmatrix} = \frac{\alpha_t}{2\pi} \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} \Delta m_t^2 + \frac{\alpha_b}{2\pi} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \Delta m_b^2 \ , \tag{18}$$

with

$$\Delta m_t^2 = \delta m_{h_u}^2 + \delta m_{U_3}^2 + \delta m_{Q_3}^2 \ , \tag{19}$$

$$\Delta m_b^2 = \delta m_{h_d}^2 + \delta m_{D_3}^2 + \delta m_{Q_3}^2 \ , \tag{20}$$

where, δm^2's are RGE running parts of the bulk mass terms. The initial values of these parameters are obviously $(\delta m_{h_u}^2, \delta m_{U_3}^2, \delta m_{Q_3}^2, \delta m_{D_3}^2, \delta m_{h_d}^2) = (c_{h_u} m_0^2, c_U m_0^2, c_Q m_0^2, c_D m_0^2, c_{h_d} m_0^2)$. We evaluate these contributions numerically. In our framework, m_0 is introduced at the GUT scale of the gauge coupling constants $M_X \sim 2 \times 10^{16}$ (GeV) [13]. The RG-variable t is dimensionless and defined as $t = \ln(\sqrt{s}/M_X)$ with the energy scale $\sqrt{s}$. Additionally, in the gaugino sector, we take into account the next-to-leading order radiative corrections and the weak scale threshold correction [11].

3 Numerical Analyses

By making use of the routine *FeynHiggs* [2], we calculate the Higgs mass up to the full 2-loop level. As the corrections to the Higgs potential, up to 1-loop order of the top and bottom corrections [14] are taken into account. The Higgs mixing parameter μ and the bilinear coupling constant B of the Higgs potential are given by following equations:

$$|\mu|^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2 \beta - \frac{1}{2} m_Z^2 (\tan^2 \beta - 1) + \Delta_{1-loop}^{(1)}}{\tan^2 \beta - 1 + \Delta_{1-loop}^{(2)}} > 0 \ , \tag{21}$$

$$B = \frac{(m_{h_d}^2 + m_{h_u}^2 + 2|\mu|^2) \sin 2\beta}{2|\mu| \mathrm{sgn}(\mu)} + \Delta_{1-loop}^{(3)} \ , \tag{22}$$

where, m_{h_u} and m_{h_d} are the Higgs mass parameters given by Eqs.(12) and (13), respectively. $\Delta_{1-loop}^{(i)}$, (i=1,2, and 3) imply the 1-loop correction terms [14]. The positivity of $|\mu|^2 > 0$ certifies the appropriate shape of the Higgs potential. $\mathrm{sgn}(\mu)$, the sign of μ is fixed to positive (see subsection 3.1). Moreover, CP-odd Higgs mass is also corrected [14] up to 1-loop order with top and bottom contributions. The theoretical value of the muon g-2 in AMSB models is evaluated up to 1-loop order both of the chargino and neutralino contributions [15]. We assume that the CP-violating phase effect is negligible.

3.1 Parameter Space

In this subsection, the allowed parameter areas on the $m_0 - m_{3/2}$ plane [16] are estimated in mAMSB model given by Eq.(17). They are shown in Figs. 1. Note that a_μ^{SUSY} is redefined as $a_\mu^{SUSY} \rightarrow a_\mu^{SUSY} \times 10^{-10}$ for convenience. We fixed $\mathrm{sgn}(\mu)$ to positive, because we find no region on the parameter space with negative μ as far as a_μ^{SUSY} is positive. In Figs. 1, the left hand sides are excluded by the experimental limits of the squark and slepton masses [4, 17]. Moreover, the lower regions are restricted from the Higgs potential condition given by Eq.(21), and the mass limits of the charginos and neutralinos [4]. The contours of the Higgs mass are illustrated by the thin solid curves. The SUSY contributions of g-2 are drawn with the thick solid curves for a_μ^{SUSY}=50, 40, 30, 20, and 10, respectively. Once reliable a_μ^{SUSY} like Eq.(4) is given by the experiments, only a small region of the corresponding narrow belt is allowed as the parameter space. In another words, the allowed parameter space on $m_0 - m_{3/2}$ plane is strictly restricted from the experiments.

3.2 Upper bound of Higgs mass

The upper bound of the Higgs mass is evaluated in the allowed parameter space where is strictly restricted from the muon g-2 in this subsection. In the first step, we scan on the allowed parameter region constrained by the g-2 analysis in the previous subsection. In the next step, we find out the parameter point to maximize the Higgs mass bound on the allowed region. In the last step, we plot those maximal values of the Higgs mass bound as functions of $\tan\beta$ as shown in Fig. 2(a). Additionally, the lower bound is also obtained in the similar procedure. For instance, $\tan\beta = 3$ gives $m_h > 102$ (GeV), or $m_h > 115$ (GeV) is derived when $\tan\beta \geq 10$.

3.3 Other AMSB models

In this subsection, other possibilities rather than the minimal model Eq.(17) of the bulk mass contributions are considered. Since naive AMSB models have a common serious problem that the squares of the slepton masses are negative, several scenarios are proposed to recover this defect. One of them is the gaugino assisted AMSB model [18]. This model contains the coefficients of the bulk mass as follows:

$$c_Q = 21/10, \ c_U = 8/5, \ c_D = 7/5, \ c_L = 9/10, \ c_E = 3/5, \ c_{h_u} = 9/10, \ c_{h_d} = 9/10 \ . \tag{23}$$

Another proposal is the extra U(1) model [19], and this model suggests following conditions:

$$c_Q = 3, \ c_U = -1, \ c_D = -1, \ c_L = 1, \ c_E = 1, \ c_{h_u} = -2, \ c_{h_d} = -2 \ . \tag{24}$$

We also analyze these two models in the similar way to the previous subsection. Figure 2(b) shows the upper bounds of the Higgs mass in these models in the same procedure with the previous subsection. Nonetheless, we cannot find so large difference on the upper bounds among these models as shown in Fig. 2(b).

More generally, the coefficients can be shifted freely from 1 as far as the bulk mass terms certify the positivity of the squared scalar masses. In order to evaluate the effects on the Higgs mass bound of their free shifts from 1, we move one of $c_Q, c_U, c_D, c_L, c_E, c_{h_u}$, or c_{h_d} from 1 (and all of the others are kept equal to 1). $\tan\beta = 45$ and $a_\mu^{SUSY}=10$ are fixed, because this condition is the strictest to make the upper bound of the Higgs mass largest. As Fig. 3 shows, the coefficients except c_L and c_E give not so large contribution to the upper bound of the Higgs mass. Here is a note on the c_Q and c_U. We found in our preliminary stage that their shifts appear to be sensitively increasing the upper bound if they are less than -3 . However, we also find their sensitivities are canceled with the conditions on the mass parameters of the other scalars not to acquire the VEVs as the charge and color breaking [20]. Therefore, the upper bound of the Higgs mass is not so increasing by the free shift of c_Q or c_U. The shifts of c_L and c_E from 1 contribute only to decrease the upper bound as shown in Fig. 3 . Note that c_L and c_E must be positive because these bulk mass terms are introduced to make the squared slepton masses positive.

As the result, we conclude that $m_h < 123$ (GeV) in almost AMSB models for all of $\tan\beta <60$.

4 Summary and Conclusion

We estimate the upper bound of the Higgs mass in several anomaly mediated supersymmetry breaking models. Consequently, we find $m_h < 123$ (GeV) in AMSB models. Heretofore, the upper bound of the Higgs mass has been estimated by loosely assuming some of the SUSY parameters. However, we achieve to confine the allowed parameter space into the restricted area on the $m_0 - m_{3/2}$ plane, and estimate the upper bound without assuming the parameters except $\tan\beta$ by making use of the restrictions from recent muon g-2 measurements. Therefore, we obtain a reliable estimation on the upper bound. Needless to say, $m_h = 123$ (GeV) as the upper bound of the Higgs mass is sufficiently small at present or in the next generation of the high-energy colliders. We hope to be found the Higgs boson in the near future.

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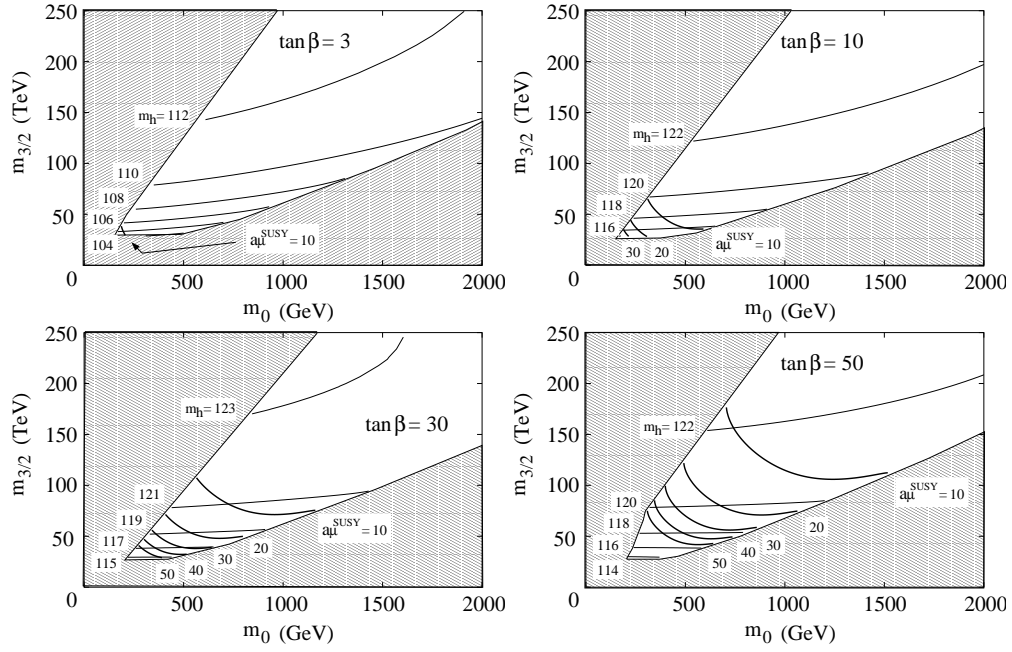
Figure 1: $m_0 - m_{3/2}$ planes with the Higgs mass (GeV) and $a_\mu^{SUSY}(\times 10^{-10})$ contours at several $\tan\beta$ in the mAMSB model. The upper left sides are excluded by the limits of the squarks and sleptons masses. Lower regions are excluded by the proper Higgs potential condition and the limits of the chargino and neutralino masses.
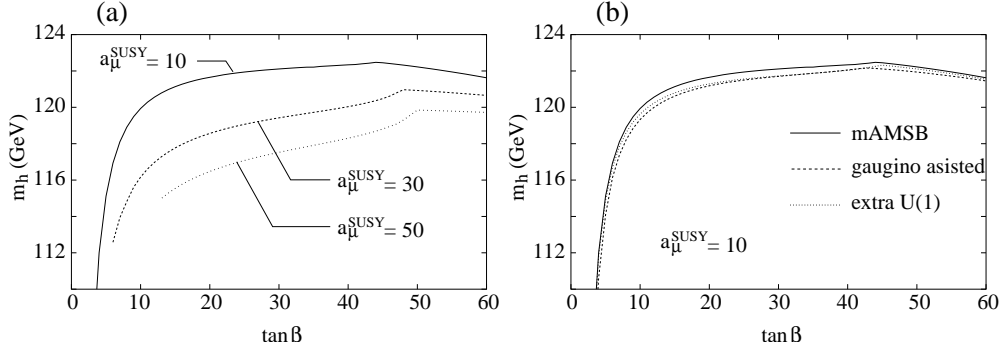

Figure 2: (a) The upper bounds of the Higgs mass depending on $\tan\beta$ for a_μ^{SUSY}=10, 30, and 50 in the mAMSB. (b) The upper bounds of the Higgs mass as functions of $\tan\beta$ in three AMSB models. a_μ^{SUSY} is fixed to 10 .
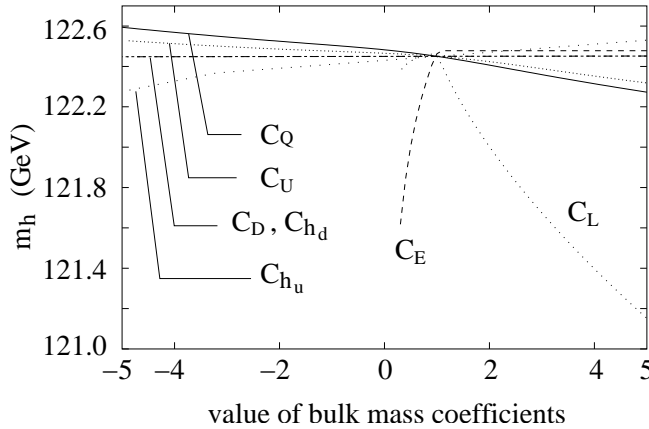

Figure 3: Effects on the Higgs mass bound from the bulk mass coefficients. Only one of these seven coefficients is moved, and all of the others are fixed to 1 . a_μ^{SUSY}=10 and $\tan\beta$=45 are fixed.